OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4413
Automatic Control Systems
Spring 2005

Midterm Exam #2

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:
The equations that describe the dynamics of a motor control system are

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta_m(t)}{dt}$$

$$T_m(t) = K_i i_a(t)$$

$$T_m(t) = J \frac{d^2\theta_m(t)}{dt^2} + B \frac{d\theta_m(t)}{dt} + K\theta_m(t)$$

$$e_a(t) = K_a e(t)$$

$$e(t) = K_s\left[\theta_r(t) - \theta_m(t)\right]$$

a) Assign the state variables as $x_1(t) = \theta_m(t)$, $x_2(t) = d\theta_m(t)/dt$, and $x_3(t) = i_a(t)$. Express the state space representation in the form of

$$\frac{dx(t)}{dt} = Ax(t) + B\theta_r(t), \quad \theta_m(t) = Cx(t).$$

b) Find the transfer function $G(s) = \Theta_m(s)/E(s)$ when the feedback path from $\Theta_m(s)$ to $E(s)$ is broken. Find the closed-loop transfer function, $M(s) = \Theta_m(s)/\Theta_r(s)$.

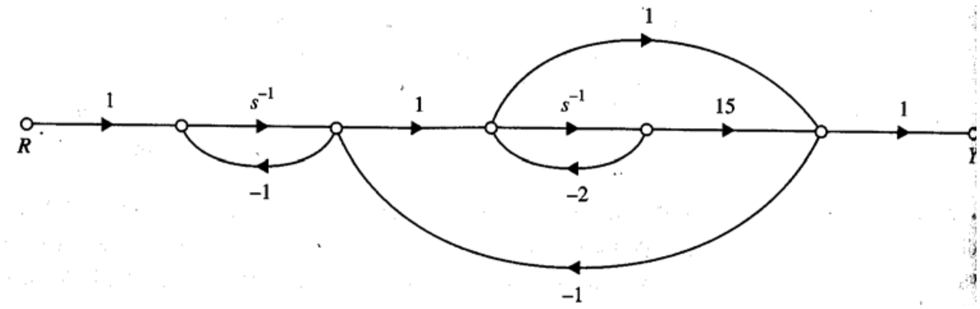
Problem 2:
For the matrices

$$A_1 = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

determine the functions of matrices $e^{A_1 t}$ and A_2^{99}.

Problem 3:

Derive a state space representation of the system given in the state diagram shown below in the form of $\dot{x}(t) = Ax(t) + Br(t), \quad y(t) = Cx(t)$,



<u>Problem 4</u>:

Find an "equivalent" *Jordan-canonical-form* dynamical equation of

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ -1 & 1 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} x(t).$$

<u>Problem 5</u>:

Show that two equivalent systems through similarity transformation are

a) zero-state equivalent (i.e., $Ce^{A(t-\tau)}B + D\delta(t-\tau) = \overline{C}e^{\overline{A}(t-\tau)}\overline{B} + \overline{D}\delta(t-\tau)$), and

b) zero-input equivalent (i.e., $Ce^{A(t-t_0)}x(t_0) = \overline{C}e^{\overline{A}(t-t_0)}\overline{x}(t_0)$).